

The seesaw mechanism in the $\mu\nu$ SSM

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The $\mu\nu$ SSM proposes to use right-handed neutrino supermultiplets in order to generate the μ term and neutrino masses simultaneously. We discuss neutrino physics and the associated electroweak seesaw mechanism in this model. We show how to obtain, from the neutralino-neutrino mass matrix of the $\mu\nu$ SSM, the effective neutrino mass matrix. In particular we discuss certain limits of this matrix that clarify the neutrino-sector behavior of the model. We also show that current data on neutrino masses and mixing angles can easily be reproduced. These constraints can be fulfilled even with a diagonal neutrino Yukawa matrix, since this seesaw does not involve only the right-handed neutrinos but also the MSSM neutralinos. To obtain the correct neutrino angles turns out to be easy due to the following characteristics of this seesaw: R-parity is broken and the relevant scale is the electroweak one.

Keywords: Supersymmetry Phenomenology; Neutrino Physics; Beyond the Standard Model.

1. Introduction

The “ μ from ν ” Supersymmetric Standard Model ($\mu\nu$ SSM) was proposed in Ref. 1 as an alternative to the Minimal Supersymmetric Standard Model (MSSM). In particular, it provides a solution to the μ -problem ² of the MSSM and explains the origin of neutrino masses by simply using right-handed neutrino superfields.

Since it was proposed ¹, the $\mu\nu$ SSM has been studied in Ref. 3 -8 (for a summary see Ref. 9). The superpotential of the $\mu\nu$ SSM is given by¹:

$$W = \epsilon_{ab} \left(Y_u^{ij} \hat{H}_u^b \hat{Q}_i^a \hat{u}_j^c + Y_d^{ij} \hat{H}_d^a \hat{Q}_i^b \hat{d}_j^c + Y_e^{ij} \hat{H}_d^a \hat{L}_i^b \hat{e}_j^c + Y_\nu^{ij} \hat{H}_u^b \hat{L}_i^a \hat{\nu}_j^c \right) \\ - \epsilon_{ab} \lambda^i \hat{\nu}_i^c \hat{H}_d^a \hat{H}_u^b + \frac{1}{3} \kappa^{ijk} \hat{\nu}_i^c \hat{\nu}_j^c \hat{\nu}_k^c. \quad (1)$$

A Z^3 symmetry could be evoked to forbid parameters with dimensions in the superpotential. This symmetry may have a clear origin if the Lagrangian describes massless modes of a fundamental object (with very massive modes, as in string theory). Non-renormalizable contributions could lift this Z^3 symmetry avoiding a possible domain wall problem without introducing hierarchy problems, in a similar way as in the NMSSM case ¹⁰.

Working in the framework of gravity mediated supersymmetry breaking, the following soft terms appear in the Lagrangian ¹,

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} = & (m_Q^2)^{ij} \tilde{Q}_i^{a*} \tilde{Q}_j^a + (m_{\tilde{u}^c}^2)^{ij} \tilde{u}_i^{c*} \tilde{u}_j^c + (m_{\tilde{d}^c}^2)^{ij} \tilde{d}_i^{c*} \tilde{d}_j^c + (m_L^2)^{ij} \tilde{L}_i^{a*} \tilde{L}_j^a + (m_{\tilde{e}^c}^2)^{ij} \tilde{e}_i^{c*} \tilde{e}_j^c \\
& + m_{H_d}^2 H_d^{a*} H_d^a + m_{H_u}^2 H_u^{a*} H_u^a + (m_{\tilde{\nu}^c}^2)^{ij} \tilde{\nu}_i^{c*} \tilde{\nu}_j^c \\
& + \epsilon_{ab} \left[(A_u Y_u)^{ij} H_u^b \tilde{Q}_i^a \tilde{u}_j^c + (A_d Y_d)^{ij} H_d^a \tilde{Q}_i^b \tilde{d}_j^c + (A_e Y_e)^{ij} H_d^a \tilde{L}_i^b \tilde{e}_j^c \right. \\
& + (A_\nu Y_\nu)^{ij} H_u^b \tilde{L}_i^a \tilde{\nu}_j^c - (A_\lambda \lambda)^i \tilde{\nu}_i^c H_d^a H_u^b + \text{H.c.} \left. \right] + \left[\frac{1}{3} (A_\kappa \kappa)^{ijk} \tilde{\nu}_i^c \tilde{\nu}_j^c \tilde{\nu}_k^c + \text{H.c.} \right] \\
& - \frac{1}{2} \left(M_3 \tilde{\lambda}_3 \tilde{\lambda}_3 + M_2 \tilde{\lambda}_2 \tilde{\lambda}_2 + M_1 \tilde{\lambda}_1 \tilde{\lambda}_1 + \text{H.c.} \right). \tag{2}
\end{aligned}$$

Let us recall that the only scale in the model is the SUSY breaking scale present in the above soft terms ¹.

Once the electroweak symmetry is spontaneously broken, the neutral scalars develop in general the following VEVs, $\langle H_d^0 \rangle = v_d$, $\langle H_u^0 \rangle = v_u$, $\langle \tilde{\nu}_i \rangle = \nu_i$, $\langle \tilde{\nu}_i^c \rangle = \nu_i^c$. An effective μ term, $\mu \equiv \lambda_i \nu_i^c$, is generated from $\lambda^i \tilde{\nu}_i^c \hat{H}_d \hat{H}_u$ in the superpotential. In a similar way terms of the type $\hat{\nu}^c \hat{\nu}^c \hat{\nu}^c$ are generating an effective Majorana mass $\sim \kappa \nu_i^c$ of electroweak order. Note that the presence at the same time of the Yukawa term for the neutrino and the last two terms in (1) are breaking R-parity explicitly.

Since in this model R-parity is explicitly broken, the neutralino is not a possible candidate for dark matter (and neither is the sneutrino). In this respect the gravitino as a dark matter candidate was studied in Ref. 5, where a possible detection was discussed.

The breaking of R-parity can easily be understood if we realize that in the limit where Yukawas for neutrinos are vanishing, the $\hat{\nu}^c$ are just ordinary singlet superfields, without any connection with neutrinos, and this model would coincide (although with three instead of one singlet) with the Next-to-Minimal Supersymmetric Standard Model (NMSSM) where R-parity is conserved.

Once we switch on the neutrino Yukawa couplings, the fields $\hat{\nu}^c$ become right-handed neutrino superfields, and, as a consequence, R-parity is broken. Indeed this breaking is small because we have an electroweak-scale seesaw, implying neutrino Yukawa couplings no larger than 10^{-6} (like the electron Yukawa) ¹.

Note that in principle the number of right-handed neutrinos is a free parameter. To fix the number we have followed in this work the philosophy that three families for all leptons is natural. With respect to this, the characteristics of the $\mu\nu$ SSM with different number of neutrinos have been discussed in Ref. 7, where the case of only one right-handed neutrino is specially discussed. Regarding the case of two right-handed neutrinos, which is the minimal number to generate the experimental pattern at tree level, it was pointed out in Ref. 4 that two is also the minimal number to have the possibility of Spontaneous CP Violation (SCPV).

In the following we describe the seesaw mechanism in the $\mu\nu$ SSM.

2. The seesaw mechanism in the $\mu\nu$ SSM

In the $\mu\nu$ SSM the MSSM neutralinos mix with the left- and right-handed neutrinos as a consequence of R-parity violation. Therefore the right-handed neutrinos behave as singlino components of the neutralinos. In the basis $\chi^{0T} = (\tilde{B}^0, \tilde{W}^0, \tilde{H}_d, \tilde{H}_u, \nu_{R_i}, \nu_{L_i})$ the neutralino-neutrino mass matrix was given in Refs. 1 and 3 for real VEVs, and in Ref. 4 for complex ones. For simplicity we work in the real case, and then we have,

$$\mathcal{M}_n = \begin{pmatrix} M & m \\ m^T & 0_{3 \times 3} \end{pmatrix}, \quad (3)$$

where the mass matrix, M , is,

$$\begin{pmatrix} M_1 & 0 & -A\langle H_d^0 \rangle & A\langle H_u^0 \rangle & 0 & 0 & 0 \\ 0 & M_2 & B\langle H_d^0 \rangle & -B\langle H_u^0 \rangle & 0 & 0 & 0 \\ -A\langle H_d^0 \rangle & B\langle H_d^0 \rangle & 0 & -\lambda_i \langle \tilde{\nu}_i^c \rangle & -\lambda_1 \langle H_u^0 \rangle & -\lambda_2 \langle H_u^0 \rangle & -\lambda_3 \langle H_u^0 \rangle \\ A\langle H_u^0 \rangle & -B\langle H_u^0 \rangle & -\lambda_i \langle \tilde{\nu}_i^c \rangle & 0 & -\lambda_1 \langle H_d^0 \rangle + Y_{\nu_{i1}} \langle \tilde{\nu}_i \rangle & -\lambda_2 \langle H_d^0 \rangle + Y_{\nu_{i2}} \langle \tilde{\nu}_i \rangle & -\lambda_3 \langle H_d^0 \rangle + Y_{\nu_{i3}} \langle \tilde{\nu}_i \rangle \\ 0 & 0 & -\lambda_1 \langle H_u^0 \rangle & -\lambda_1 \langle H_d^0 \rangle + Y_{\nu_{i1}} \langle \tilde{\nu}_i \rangle & 2\kappa_{11j} \langle \tilde{\nu}_j^c \rangle & 2\kappa_{12j} \langle \tilde{\nu}_j^c \rangle & 2\kappa_{13j} \langle \tilde{\nu}_j^c \rangle \\ 0 & 0 & -\lambda_2 \langle H_u^0 \rangle & -\lambda_2 \langle H_d^0 \rangle + Y_{\nu_{i2}} \langle \tilde{\nu}_i \rangle & 2\kappa_{21j} \langle \tilde{\nu}_j^c \rangle & 2\kappa_{22j} \langle \tilde{\nu}_j^c \rangle & 2\kappa_{23j} \langle \tilde{\nu}_j^c \rangle \\ 0 & 0 & -\lambda_3 \langle H_u^0 \rangle & -\lambda_3 \langle H_d^0 \rangle + Y_{\nu_{i3}} \langle \tilde{\nu}_i \rangle & 2\kappa_{31j} \langle \tilde{\nu}_j^c \rangle & 2\kappa_{32j} \langle \tilde{\nu}_j^c \rangle & 2\kappa_{33j} \langle \tilde{\nu}_j^c \rangle \end{pmatrix}, \quad (4)$$

with $A = \frac{G}{\sqrt{2}} \sin \theta_W$, $B = \frac{G}{\sqrt{2}} \cos \theta_W$, and

$$m^T = \begin{pmatrix} -\frac{g_1}{\sqrt{2}} \langle \tilde{\nu}_1 \rangle & \frac{g_2}{\sqrt{2}} \langle \tilde{\nu}_1 \rangle & 0 & Y_{\nu_{1i}} \langle \tilde{\nu}_i^c \rangle & Y_{\nu_{11}} \langle H_u^0 \rangle & Y_{\nu_{12}} \langle H_u^0 \rangle & Y_{\nu_{13}} \langle H_u^0 \rangle \\ -\frac{g_1}{\sqrt{2}} \langle \tilde{\nu}_2 \rangle & \frac{g_2}{\sqrt{2}} \langle \tilde{\nu}_2 \rangle & 0 & Y_{\nu_{2i}} \langle \tilde{\nu}_i^c \rangle & Y_{\nu_{21}} \langle H_u^0 \rangle & Y_{\nu_{22}} \langle H_u^0 \rangle & Y_{\nu_{23}} \langle H_u^0 \rangle \\ -\frac{g_1}{\sqrt{2}} \langle \tilde{\nu}_3 \rangle & \frac{g_2}{\sqrt{2}} \langle \tilde{\nu}_3 \rangle & 0 & Y_{\nu_{3i}} \langle \tilde{\nu}_i^c \rangle & Y_{\nu_{31}} \langle H_u^0 \rangle & Y_{\nu_{32}} \langle H_u^0 \rangle & Y_{\nu_{33}} \langle H_u^0 \rangle \end{pmatrix}. \quad (5)$$

The above matrix (3) is of the seesaw type where the entries of M are of the order of the electroweak scale while the ones in m are of the order of the Dirac masses for the neutrinos^{1,3}. Therefore in a first approximation the effective neutrino mixing mass matrix can be written as,

$$m_{eff} = -m^T \cdot M^{-1} \cdot m, \quad (6)$$

and one can diagonalize it by a unitary transformation (ortonormal in the real case)

$$U_{MNS}^T m_{eff} U_{MNS} = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}). \quad (7)$$

Let us write an approximate analytical expression for the effective neutrino mass matrix of the $\mu\nu$ SSM neglecting all the terms containing $Y_\nu^2 \nu^2$, $Y_\nu^3 \nu$ and $Y_\nu \nu^3$, due to the smallness of Y_ν and ν ¹, and taking couplings $\lambda_i \equiv \lambda$, the tensor κ with terms $\kappa_{iii} \equiv \kappa_i \equiv \kappa$ and vanishing otherwise, diagonal Yukawa couplings $Y_{\nu_{ii}} \equiv Y_{\nu_i}$, and VEVs $\nu_i^c \equiv \nu^c$. From Ref. 4 (also this coincides with the results of Ref. 6) we have,

$$(m_{eff})_{ij} \simeq \frac{v_d^2}{6\kappa\nu^c} Y_{\nu_i} Y_{\nu_j} (1 - 3\delta_{ij}) - \frac{1}{2M_{eff}} \left[\nu_i \nu_j + \frac{v_d (Y_{\nu_i} \nu_j + Y_{\nu_j} \nu_i)}{3\lambda} + \frac{Y_{\nu_i} Y_{\nu_j} v_d^2}{9\lambda^2} \right], \quad (8)$$

with

$$M_{eff} \equiv M \left[1 - \frac{v^2}{2M(\kappa\nu^c + \lambda v_u v_d)} \frac{1}{3\lambda\nu^c} \left(2\kappa\nu^c \frac{v_u v_d}{v^2} + \frac{\lambda v^2}{2} \right) \right]. \quad (9)$$

Here $v^2 = v_u^2 + v_d^2 + \sum_i \nu_i^2 \approx v_u^2 + v_d^2 \approx (174\text{GeV})^2$ has been used, since $\nu_i \ll v_u, v_d$ ¹, and also we have defined $\frac{1}{M} = \frac{g_1^2}{M_1} + \frac{g_2^2}{M_2}$.

To clarify how the seesaw works in this model let us make three limits, making special emphasis in the last one.

The first limit^{6,4} is the one where gauginos are very heavy and decouple (i.e. $M \rightarrow \infty$), and then (8) reduces to

$$(m_{eff})_{ij} \simeq \frac{v_u^2}{6\kappa\nu^c} Y_{\nu_i} Y_{\nu_j} (1 - 3\delta_{ij}). \quad (10)$$

The effective mixing of the right-handed neutrinos and Higgsinos in this limit produces off-diagonal entries in the mass matrix. Besides, when right-handed neutrinos are also decoupled (i.e. $\nu^c \rightarrow \infty$), the neutrino masses are zero as corresponds to the case of a seesaw with only Higgsinos.

The second limit^{6,4} which is worth discussing is $\nu^c \rightarrow \infty$. Then, (8) reduces to the form

$$(m_{eff})_{ij} \simeq -\frac{1}{2M} \left[\nu_i \nu_j + \frac{v_d(Y_{\nu_i} \nu_j + Y_{\nu_j} \nu_i)}{3\lambda} + \frac{Y_{\nu_i} Y_{\nu_j} v_d^2}{9\lambda^2} \right]. \quad (11)$$

We can also see that for $v_d \rightarrow 0$ (i.e. $\tan\beta = \frac{v_u}{v_d} \rightarrow \infty$) one obtains

$$(m_{eff})_{ij} \simeq -\frac{\nu_i \nu_j}{2M}. \quad (12)$$

Note that this result can actually be obtained if $\nu_i \gg \frac{Y_{\nu_i} v_d}{3\lambda}$, as was noticed in Ref. 4. It means that decoupling right-handed neutrinos/singlinos and Higgsinos, the seesaw mechanism is generated through the mixing of left-handed neutrinos with gauginos. This is a characteristic feature of the seesaw in the well-known bilinear R-parity violation model (BRpV)¹¹. Notice that the gauginos (Bino and Wino) are in the adjoint representation, so they have a vector character, and (12) could be associated to a type III seesaw^a as (10) is associated to a Type I. Nevertheless we recall that there is an intrinsically supersymmetric character of this seesaw, since gauginos are part of gauge supermultiplets.

The last limit⁴ we want to discuss gives a clear idea of the seesaw mechanism in this model. The seesaw in the $\mu\nu$ SSM comes, in general, from the interplay of the above two limits. Namely, the limit where we suppress only certain Higgsino and gaugino mixing. Hence, taking $v_d \rightarrow 0$ in (8), we obtain

$$(m_{eff})_{ij} \simeq \frac{v_u^2}{6\kappa\nu^c} Y_{\nu_i} Y_{\nu_j} (1 - 3\delta_{ij}) - \frac{1}{2M_{\text{eff}}} \nu_i \nu_j, \quad (13)$$

As above, we remark that actually this result can be obtained if $\nu_i \gg \frac{Y_{\nu_i} v_d}{3\lambda}$. The effective mass $M_{\text{eff}} = M \left(1 - \frac{v^4}{12\kappa M \nu^c} \right)$ represents the mixing between gauginos and Higgsinos- ν^c that is not completely suppressed in this limit. Expression (13) is more general than the other two limits studied above. Notice that for typical values of the parameters involved in the seesaw, $M_{\text{eff}} \approx M$, and therefore we get

^aThe association of (12) to a type III seesaw was pointed out in Ref. 8.

a simple formula that can be used to understand the seesaw mechanism in this model in an qualitative way, that is

$$(m_{eff})_{ij} \simeq \frac{v_u^2}{6\kappa\nu^c} Y_{\nu_i} Y_{\nu_j} (1 - 3\delta_{ij}) - \frac{1}{2M} \nu_i \nu_j. \quad (14)$$

The simplicity of (14), in contrast with the full formula given by (8), comes from the fact that the mixing between gauginos and Higgsinos- ν^c is neglected. In this model the effective neutrino mass matrix is not diagonal even for diagonal Yukawas. Following Ref. 4, in the next section we show how to obtain the normal and inverted hierarchy, reproducing experimental masses and mixing angles, in an easy way, using diagonal Yukawas ^b.

3. Normal and Inverted hierarchies obtained with diagonal Yukawas

To continue the discussion of the seesaw in the $\mu\nu$ SSM, let us remind that two mass differences and mixing angles have been measured experimentally in the neutrino sector. The allowed 3σ ranges for these parameters are shown in Table 1 (as discussed in Ref. 12). We also show the compositions of the mass eigenstates in Fig. 1 for the normal and inverted hierarchy cases (as shown in Ref. 13).

Table 1. Allowed 3σ ranges for the neutrino masses and mixings.

$\Delta m_{sol}^2/10^{-5} \text{ eV}^2$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{13}$	$\sin^2 \theta_{23}$	$\Delta m_{atm}^2/10^{-3} \text{ eV}^2$
7.14-8.19	0.263-0.375	< 0.046	0.331-0.644	2.06-2.81

Due to the fact that the mass eigenstates have, in a good approximation, the same composition of ν_μ and ν_τ we start considering diagonal and degenerate Yukawas in μ and τ , $Y_{\nu_2} = Y_{\nu_3}$, also with $\nu_2 = \nu_3$. Therefore (14) takes the form

$$m_{eff} = \begin{pmatrix} d & c & c \\ c & A & B \\ c & B & A \end{pmatrix}, \quad (15)$$

where

$$\begin{aligned} d &= -\frac{v_u^2}{3\kappa\nu^c} Y_{\nu_1}^2 - \frac{1}{2M} \nu_1^2, \quad c = \frac{v_u^2}{6\kappa\nu^c} Y_{\nu_1} Y_{\nu_2} - \frac{1}{2M} \nu_1 \nu_2, \\ A &= -\frac{v_u^2}{3\kappa\nu^c} Y_{\nu_2}^2 - \frac{1}{2M} \nu_2^2, \quad B = \frac{v_u^2}{6\kappa\nu^c} Y_{\nu_2}^2 - \frac{1}{2M} \nu_2^2. \end{aligned} \quad (16)$$

The eigenvalues of this matrix are the following:

$$\frac{1}{2} \left(A + B - \sqrt{8c^2 + (A + B - d)^2} + d \right), \frac{1}{2} \left(A + B + \sqrt{8c^2 + (A + B - d)^2} + d \right), A - B, \quad (17)$$

and the corresponding eigenvectors (for simplicity are not normalised) are

$$\left(-\frac{A + B + \sqrt{8c^2 + (A + B - d)^2} - d}{2}, c, c \right), \left(\frac{-A - B + \sqrt{8c^2 + (A + B - d)^2} + d}{2c}, 1, 1 \right), (0, -1, 1). \quad (18)$$

^bThe first solutions for masses and mixing angles in the experimental allowed range using diagonal Yukawas were found in Ref. 6.

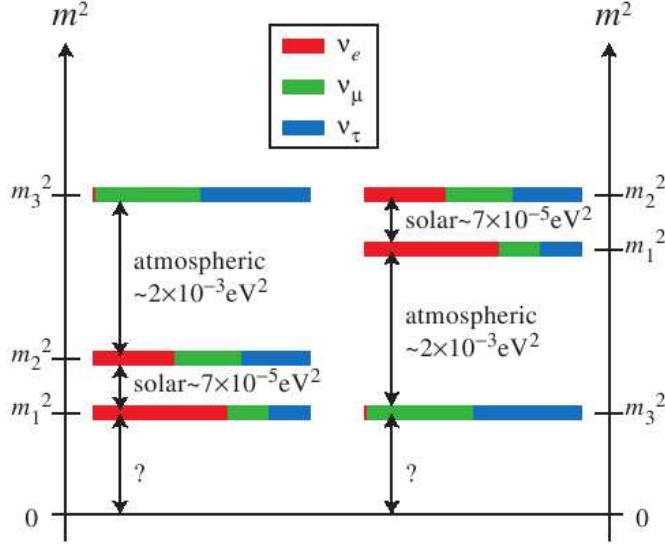


Fig. 1. The two possible hierarchies of neutrino masses. The pattern on the left side corresponds to the normal hierarchy and is characterized by one heavy state with a very little electron neutrino component, and two light states with a mass difference which is the solar mass difference. The pattern on the right side corresponds to the inverted hierarchy and is characterized by two heavy states with a mass difference that is the solar mass difference, and a light state which has very little electron neutrino component.

We have ordered the eigenvalues in such a way that it is clear how to obtain the normal hierarchy for the ν_μ - ν_τ degenerate case. Then we see that $\sin^2 \theta_{13} = 0$ and $\sin^2 \theta_{23} = \frac{1}{2}$, as in the bimaximal mixing regime. Also we have enough freedom to fix the parameters in such a way that the experimental values for the mass differences and the remaining angle θ_{12} can be reproduced. It is important to mention that the above two values of the angles are a consequence of considering the example with ν_μ - ν_τ degeneration, and therefore valid even if we use the general formula (8) instead of the simplified expression (14). Notice that (15), (17) and (18) would be the same but with the corresponding values of A, B, c and d .

Let us finally remark that we can get the tri-bimaximal mixing regime, i.e., $\sin^2 \theta_{13} = 0$, $\sin^2 \theta_{23} = 1/2$ and $\sin^2 \theta_{12} = 1/3$, fixing in (15) $c = A + B - d$. In this way we obtain the eigenvalues

$$-(A + B) + 2d, \quad 2(A + B) - d, \quad A - B. \quad (19)$$

It is important to note that, in this case, we need $|A - B| > |2(A + B) - d|$ for the normal hierarchy case, otherwise the θ_{12} angle is zero instead of θ_{13} .

In the inverted hierarchy scenario $|A - B| > |2(A + B) - d|$ leads the angle θ_{12} to zero which is not phenomenologically viable. Then we impose $|A - B| < |2(A + B) - d|$.

In this way we obtain the eigenvalues,

$$-(A + B) + 2d = m_{\nu_1}, \quad 2(A + B) - d = m_{\nu_2}, \quad A - B = m_{\nu_3}, \quad (20)$$

Where for the normal case $|A - B| = m_{\nu_3}$ is the heaviest one and for the inverted case is the lightest

one, as in Fig 1.

Breaking the degeneracy between the Y_ν and ν for the muon and tau neutrinos, it is possible to find more general solutions in the normal and inverted hierarchy cases. Here we have shown an example to reproduce the experiment, but more possibilities can be found in Refs. 6, 4 and 8.

It is remarkable that one could assume diagonal Yukawas for the leptons, reproducing all the neutrinos masses and mixing angle. In other words, the leptonic sector of this model has the ability to give large mixing angles using a simple diagonal structure for the Yukawas. In a sense, the model gives an explanation of why the mixing angles of the quark and lepton sector are so different.

The characteristics of the seesaw in this model can easily be understood from the limit (14): R-parity is broken (neutralinos are part of the seesaw) and the only scale is the SUSY one ⁴.

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